Basics of Aerospace

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Lecture 6

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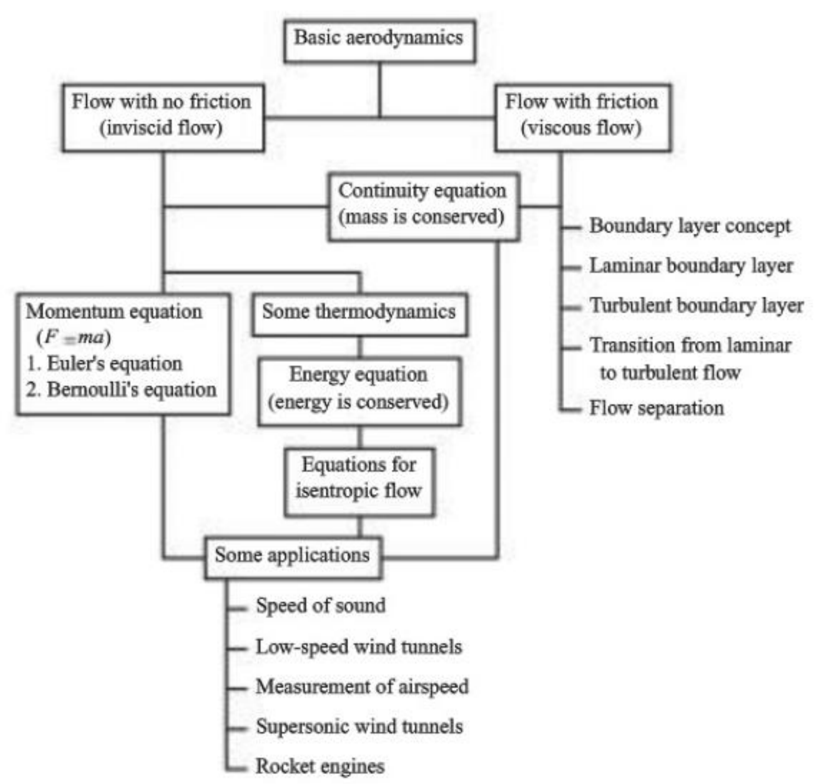
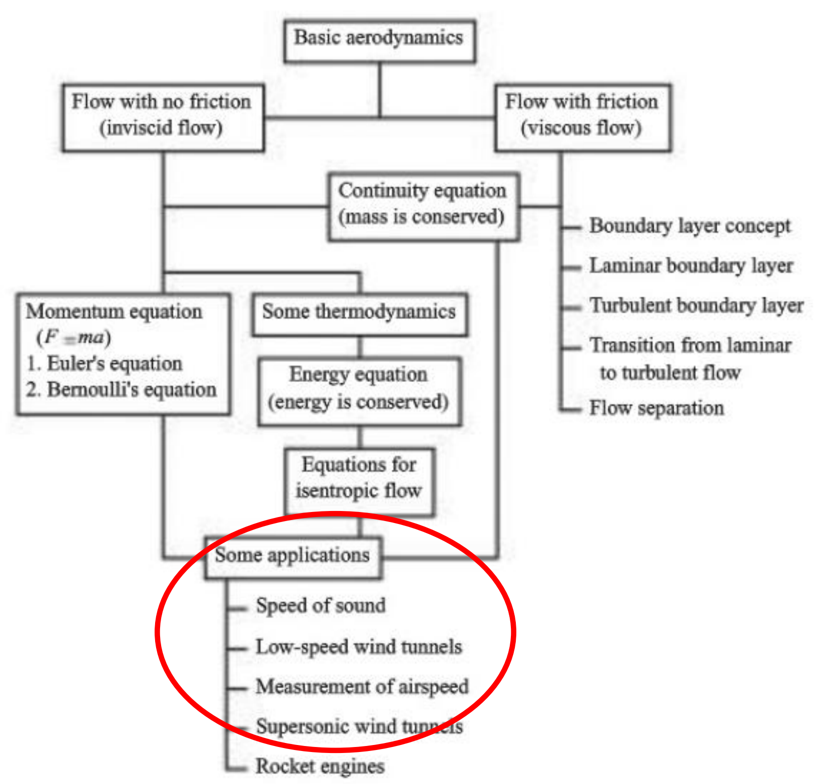
Week 6: Aerodynamics III

- Speed of Sound
- Subsonic Wind Tunnels
- Airspeed Measurement
- Why Mach > 0.3?

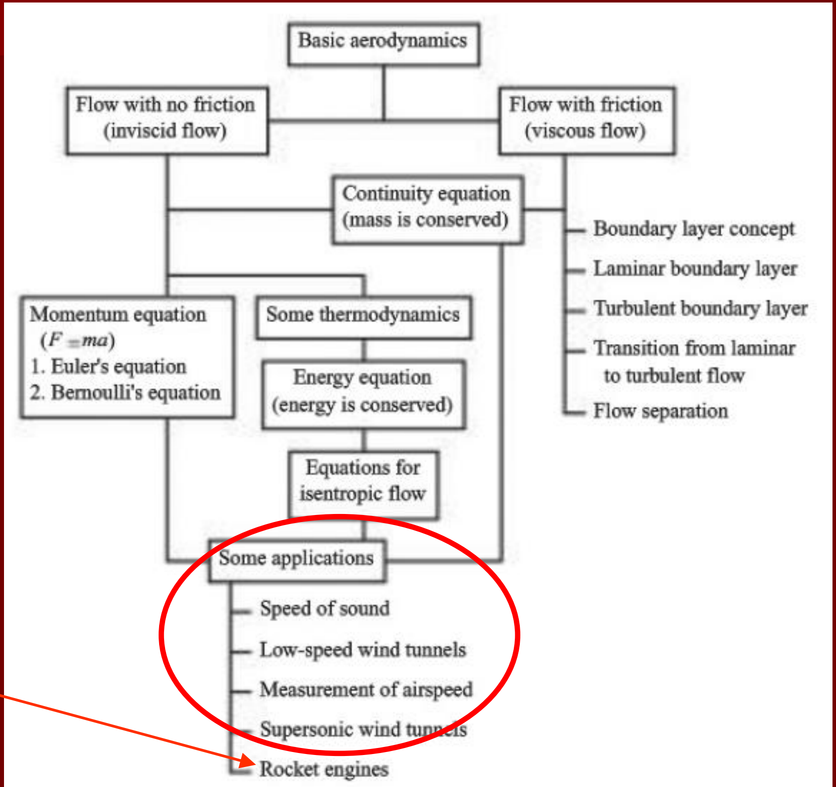
Map of Aerodynamics



Map of Aerodynamics



Map of Aerodynamics



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Speed of Sound

Ernst Mach and his number

Speed of Sound

- Sound, a mechanical wave, travels through its media with a certain speed
- It can be shown that this speed is:

$$a = \sqrt{\gamma R T}$$

Speed of Sound

- Note that:
  - *For a perfect gas, the speed of sound depends only on temperature*
  - Speed of sound is a property at a point, and therefore changes along a fluid flow

$$a = \sqrt{\gamma R T}$$

Mach Number

- From the speed of sound, we can define the *Mach Number:*

$$M = \frac{V}{a}$$

- Note once again that the Mach number is a point property: the ratio between the velocity and speed of sound *at a point*

Mach Number

- This is an incredibly important number in aerodynamics. It defines 3 flow regimes:
  - $M < 1$: Subsonic
  - $M = 1$: Sonic
  - $M > 1$: Supersonic
- The flow is also said to be *transonic* when $0.8 < M < 1.2$ and *hypersonic* when $M > 5$

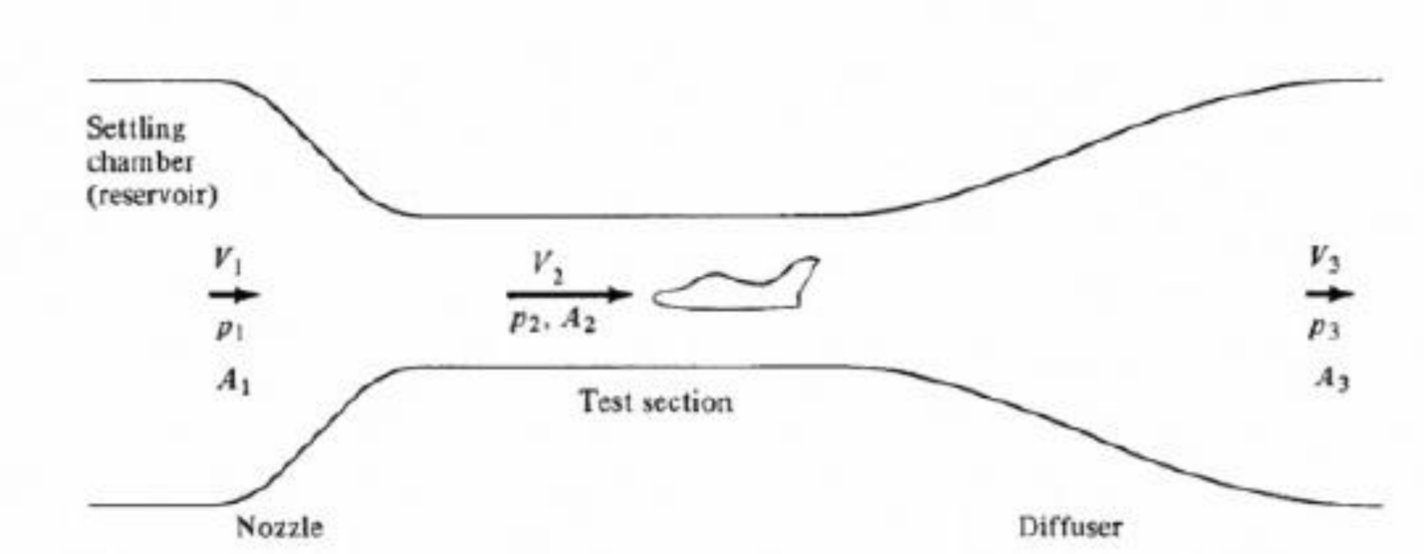
Subsonic Wind Tunnels

The way we test aerodynamics

Wind Tunnels

- Wind tunnels are devices that allow us to control the velocity over a section (called the *test section*) in order to conduct experiments on aerodynamics over bodies, wings, airfoils, etc.
- There are generally 3 parts to a wind tunnel: a nozzle, test section, and diffuser

Subsonic Wind Tunnels



Subsonic Wind Tunnels

- From continuity and assuming incompressible flow:

$$V_2 = \frac{A_1}{A_2} V_1 = \frac{A_3}{A_2} V_3$$

- But Bernoulli's:

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2 = p_3 + \frac{1}{2}\rho V_3^2$$

Subsonic Wind Tunnels

- In most tunnels, p_2 is "vented" to atmospheric pressure to allow easy access into and out of the tunnel
- Also, the ratio $\frac{A_3}{A_1} \cong 1$ such that $V_1 = V_3$
- In these tunnels, the operation is governed by the pressure difference $p_2 - p_1$ and $\frac{A_2}{A_1}$

Subsonic Wind Tunnels

- From Bernoulli's:

$$V_2^2 = \frac{2}{\rho}(p_1 - p_2) + V_1^2$$

- But $V_1 = \frac{A_2}{A_1} V_2$:

$$V_2^2 = \frac{2}{\rho}(p_1 - p_2) + \left(\frac{A_2}{A_1}\right)^2 V_2^2$$

Subsonic Wind Tunnels

- Solving for V_2 yields:

$$V_2 = \sqrt{\frac{2(p_1 - p_2)}{\rho[1 - (A_2/A_1)^2]}}$$

- Where the tunnel's "control knob" adjusts $p_1 - p_2$ and $\frac{A_2}{A_1}$ is set by the designer

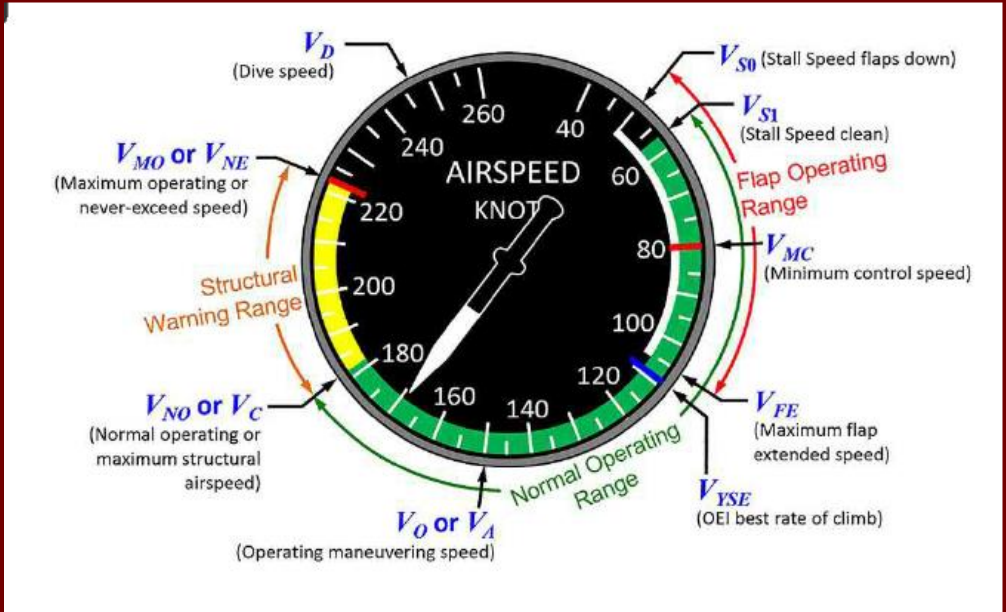
Airspeed Measurement

Airspeed is life

Airspeed

- Airspeed is the relative velocity of air with respect to some medium, usually the aircraft (or wing, airfoil, etc.)
- Airspeed is perhaps the most crucial aerodynamic piece of information available to the pilot

Airspeed

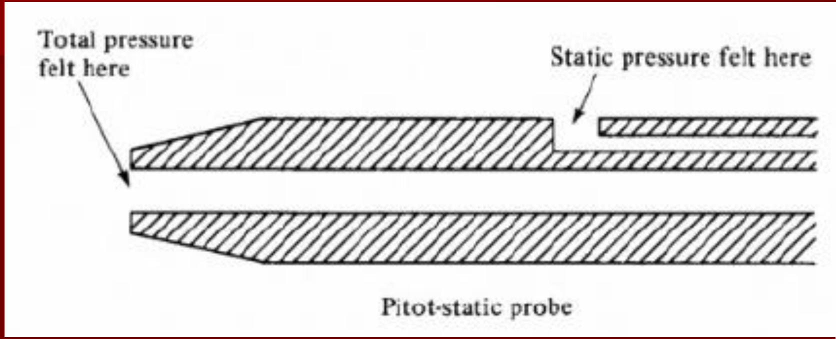


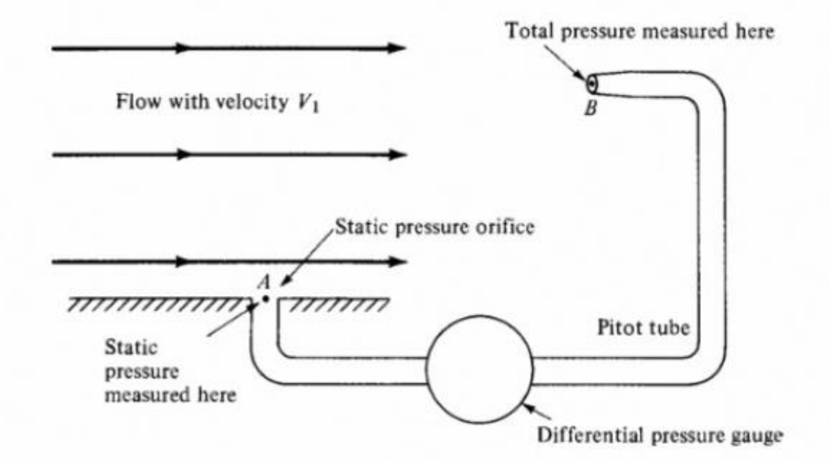
V_D
(Dive speed)
V_{S0} (Stall Speed flaps down)
V_{S1}
(Stall Speed clean)
V_{MO} or V_{NE}
(Maximum operating or never-exceed speed)
Flap Operating Range
AIRSPEED
KNOTS
260
40
240
60
220
80
200
100
180
120
160
140
V_{MC}
(Minimum control speed)
Structural Warning Range
V_{NO} or V_C
(Normal operating or maximum structural airspeed)
V_{FE}
(Maximum flap extended speed)
Normal Operating Range
V_{YSE}
(OEI best rate of climb)
V_O or V_A
(Operating maneuvering speed)

Subsonic Incompressible Flow

- Let us consider the science of airspeed measurement for subsonic incompressible flow
- Airspeed is usually measured using a device called the Pitot tube

Subsonic Incompressible Flow





Subsonic Incompressible Flow

- Recall that total pressure is:

$$p = p_0 + \frac{1}{2}\rho V^2$$

- Solving for V:

$$V = \sqrt{\frac{2(p - p_0)}{\rho}}$$

Subsonic Compressible Flow

- The compressible case is much more complex. Bear with me!
- Thermodynamics shows:

$$c_p = R + c_v$$

- From which one can find:

$$c_p = \frac{\gamma R}{\gamma - 1}$$

Subsonic Compressible Flow

- Consider a Pitot-tube. In the freestream we have T_1, p_1 (static pressure), and V_1. At the stagnation point, we have T_0, p_0 (total pressure), but $V_0 = 0$
- Recall the energy equation:

$$c_p T_1 + \frac{V_1^2}{2} = c_p T_0$$

Subsonic Compressible Flow

- Rearranging:

$$\frac{T_0}{T_1} = 1 + \frac{V_1^2}{2c_p T_1}$$

- Subbing c_p, a appears naturally:

$$\frac{T_0}{T_1} = 1 + \frac{\gamma - 1}{2} M_1^2$$

Subsonic Compressible Flow

- Recall for isentropic flow:

$$\frac{p_0}{p_1} = \left(\frac{T_0}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

- Substituting and solving for Mach yields:

$$M_1^2 = \frac{2}{\gamma - 1}\left[\left(\frac{p_0}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

Subsonic Compressible Flow

$$M_1^2 = \frac{2}{\gamma - 1}\left[\left(\frac{p_0}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

- This is an interesting result: **a measurement of p_0 (total pressure) and p_1 (static pressure) is a direct measurement of Mach number!**

Subsonic Compressible Flow

- Solving for actual velocity:

$$V_1^2 = \frac{2a_1^2}{\gamma - 1}\left[\left(\frac{p_0}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

True Airspeed

- Rearranging the expression:

$$V_1^2 = \frac{2a_1^2}{\gamma - 1}\left[\left(\frac{p_0 - p_1}{p_1} + 1\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

- These two equations are used to find the actual airspeed, the *true airspeed*

True Airspeed

- However, this requires finding a_1, which requires measuring T_1
- As it turns out, measuring T_1 is quite hard
- This + the fact that pressure gauges are set up to measure pressure *differences* $(p_0 - p_1)$ instead of *ratios* (p_0/p_1), makes it convenient to define a *calibrated airspeed*

Calibrated Airspeed

- Rearranging the expression:

$$V_{cal}^2 = \frac{2a_s^2}{\gamma - 1}\left[\left(\frac{p_0 - p_1}{p_s} + 1\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

- Where a_s and p_s are the easily measured/known speed of sound and pressure at sea-level

Why Mach > 0.3?

A discussion of compressibility

Why Mach > 0.3?

- It should be engrained in your head that any flow where $M > 0.3$ is fast enough to consider compressibility effects
- Why is this the case?

Why Mach > 0.3?

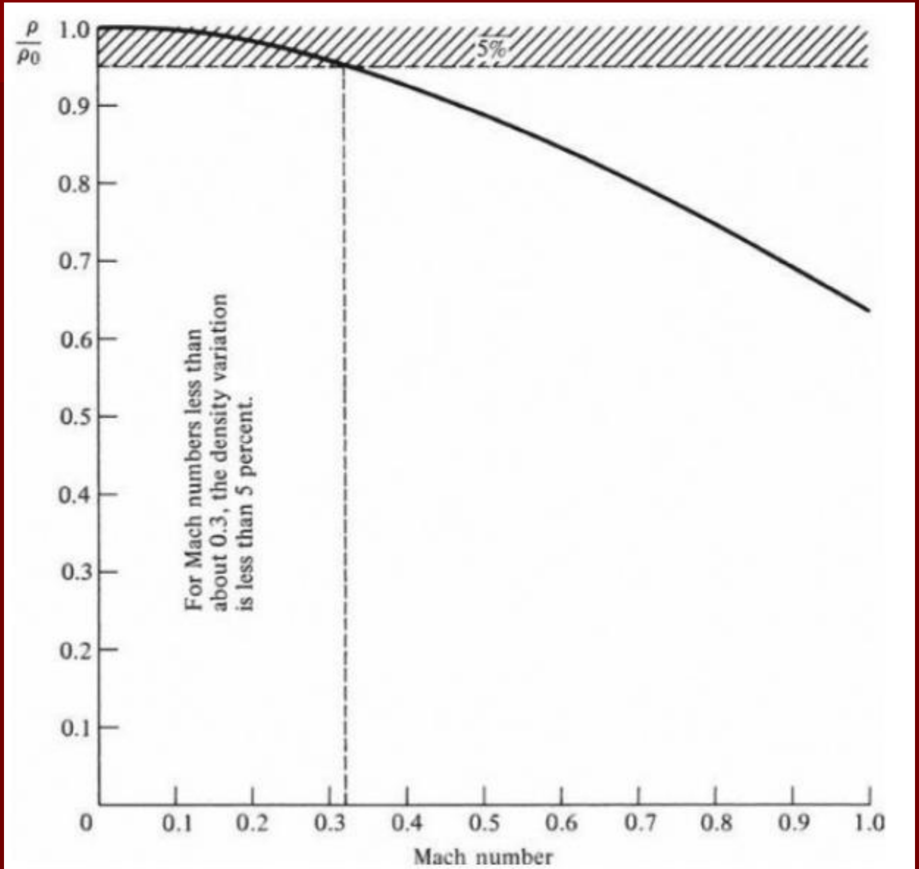
- Consider a gas initially at rest that is isentropically accelerated to Mach M
- The density ratio is given by the isentropic equations derived before:

$$\frac{\rho_0}{\rho} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{1}{\gamma - 1}}$$

Why Mach > 0.3?

- Plotting the density ratio $\frac{\rho_0}{\rho}$ versus Mach number yields the following plot:

Why Mach > 0.3?



$\frac{\rho}{\rho_0}$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
5%
For Mach numbers less than
about 0.3, the density variation
is less than 5 percent.
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Mach number

Why Mach > 0.3?

- For M < 0.3, the density variation is less than 5%
- We thus define 5% as a density variation large enough to raise the need to account for compressibility effects